

Boundary of two mixed Bose-Einstein condensates

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The boundary of two mixed Bose-Einstein condensates interacting repulsively was considered in the case of spatial separation at zero temperature. Analytical expressions for density distribution of condensates were obtained by solving two coupled nonlinear Gross-Pitaevskii equations in cases corresponding weak and strong separation. These expressions allow to consider excitation spectrum of a particle confined in the vicinity of the boundary as well as surface waves associated with surface tension.

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I. INTRODUCTION

The experimental realization of Bose Einstein condensation in trapped dilute gases [1–3] has allowed to investigate variety of properties of quantum fluids both theoretically and experimentally. In the recent years, it has become possible to produce and explore mixtures of Bose-Einstein condensates corresponding to different internal states [4–6].

Theoretical treatment of mixtures [7–10] assures that depending on the relative strength of interactions inside each condensate and between them it is possible to observe spatial separation. Experimental realization of such systems [4–6] has given an opportunity to study both equilibrium properties and dynamics of separation. Although, structure of the boundary has been qualitatively analyzed in [7,8] an analytical distribution of densities has not been derived yet. Authors of [8] discussed asymptotic behavior of densities far from the boundary, and on the basis of these estimations analyzed surface tension.

In the present paper, we explore the boundary between two repulsively interacting condensates at zero temperature in two limit cases corresponding weak and strong separation of the condensates. We show that in the case of weak separation it is possible to derive equations for densities, and using iteration method solve them analytically. We show that asymptotic behavior of our solution coincides with those predicted in [8]. For the sake of completeness, we also provide the solutions of these equations in the case of strong separation. Importance of the solution lies in the possibility to explore quantitatively different types of excitations on the boundary. The structure of the boundary in both cases allows to consider one-particle excitations as well as surface waves associated with the boundary. Obtained expressions for dispersion relation of surface waves can be used to explore that phenomenon experimentally.

The Hamiltonian describing the mixture of two weakly

interacting Bose-Einstein condensates can be written in the form

$$H = \sum_{i=1,2} \int d\mathbf{r} \Psi_i^\dagger \left[-\frac{\hbar^2 \nabla^2}{2m_i} + V_i(\mathbf{r}) + \frac{u_i}{2} \Psi_i^\dagger \Psi_i \right] \Psi_i + u_{12} \int d\mathbf{r} \Psi_1^\dagger \Psi_2^\dagger \Psi_1 \Psi_2 \quad (1)$$

Here, $u_i = 4\pi\hbar^2 a_i/m_i > 0$ characterizes the interaction inside each condensate, $u_{12} = 2\pi\hbar^2 a_{12}(m_1 + m_2)/(m_1 m_2) > 0$ – the intercondensate interaction, m_i – mass of a particle of each condensate, a_i , a_{12} – corresponding scattering lengths, $V_i(\mathbf{r})$ – external trapping potentials. Theoretical treatment of the mixtures [7,8] has shown that separation takes place when $u_{12}/\sqrt{u_1 u_2} > 1$.

Starting with the Hamiltonian (1) we get Gross-Pitaevskii equations for condensate wave functions:

$$i\hbar \frac{\partial \Psi_1}{\partial t} = \left(-\frac{\hbar^2 \nabla^2}{2m_1} + V_1(\mathbf{r}) + u_1 |\Psi_1|^2 + u_{12} |\Psi_2|^2 \right) \Psi_1, \\ i\hbar \frac{\partial \Psi_2}{\partial t} = \left(-\frac{\hbar^2 \nabla^2}{2m_2} + V_2(\mathbf{r}) + u_2 |\Psi_2|^2 + u_{12} |\Psi_1|^2 \right) \Psi_2 \quad (2)$$

As we are interested in studying of stationary solutions of these equations, then assuming as usual $\Psi_j \propto \exp(-i\mu_j t)$, where μ_j – chemical potentials of the condensates, we obtain two coupled nonlinear equations for densities of gases $n_i(\mathbf{r}) = |\Psi_i(\mathbf{r})|^2$:

$$\mu_1 = -\frac{\hbar^2}{2m_1} \frac{\nabla^2 \sqrt{n_1}}{\sqrt{n_1}} + V_1(\mathbf{r}) + u_1 n_1 + u_{12} n_2, \\ \mu_2 = -\frac{\hbar^2}{2m_2} \frac{\nabla^2 \sqrt{n_2}}{\sqrt{n_2}} + V_2(\mathbf{r}) + u_2 n_2 + u_{12} n_1 \quad (3)$$

These equations are essentially nonlinear, so to find solutions we need to make some simplifications. We assume that $V_1(\mathbf{r}) = V_2(\mathbf{r})$ and consider the case when the size of a boundary between condensates much less than characteristic length of the trap. In the case of a parabolic trap potential and Thomas-Fermi regime, it means that $d \ll R_{TF}$, where d is the size of the boundary, and R_{TF} is the Thomas-Fermi radius of the atomic cloud. Physically, it helps to avoid the effect of the potential on the form of the boundary. To simplify calculations further, we also suppose that separation takes place in one dimension, z . This allows us to write the system of equations (3) in the following form:

$$\mu_1 = -\frac{\hbar^2}{2m_1 \sqrt{n_1}} \frac{d^2}{dz^2} \sqrt{n_1} + u_1 n_1 + u_{12} n_2, \\ \mu_2 = -\frac{\hbar^2}{2m_2 \sqrt{n_2}} \frac{d^2}{dz^2} \sqrt{n_2} + u_2 n_2 + u_{12} n_1 \quad (4)$$

Although, there is no explicit trapping potential in these equations, we have to impose external conditions on the solutions, that implicitly take it into account. Let us

assume that separation takes place along the z direction, and the condensate with the label “1” is to the right, and the one with the label “2” is to the left of the boundary. Then, asymptotically we require:

$$\begin{aligned} n_1(z \rightarrow +\infty) &\rightarrow n_{10}, \quad n_1(z \rightarrow -\infty) \rightarrow 0, \\ n_2(z \rightarrow -\infty) &\rightarrow n_{20}, \quad n_2(z \rightarrow +\infty) \rightarrow 0, \end{aligned} \quad (5)$$

where n_{10} , n_{20} – equilibrium densities of condensates far from the boundary. Substituting these conditions in equations (4) for densities, and using the obvious condition of equilibrium we obtain

$$\begin{aligned} \mu_1 &= u_1 n_{10}, \quad \mu_2 = u_2 n_{20}, \\ P_1 &= u_1 n_{10}^2/2 = P_2 = u_2 n_{20}^2/2 \end{aligned} \quad (6)$$

Here, we used well-known expression for the pressure of a homogeneous weakly-interacting Bose gas. That condition connects equilibrium densities of condensates far from the boundary.

To reduce the number of parameters in equations (4) we notice that it is possible to exclude difference of masses by the change:

$$\begin{aligned} u_1^* &= u_1 m_1/m_2, \quad u_2^* = u_2 m_2/m_1, \\ n_1^* &= n_1 \sqrt{m_2/m_1}, \quad n_2^* = n_2 \sqrt{m_1/m_2}, \\ \mu_1^* &= u_1^* n_{10}^*, \quad \mu_2^* = u_2^* n_{20}^*, \\ m^* &= \sqrt{m_1 m_2} \end{aligned} \quad (7)$$

We get the same equations (4) and conditions (5), (6) but for quantities with stars and with the same mass m^* . To simplify notations, in the following discussion we omit stars. The generalization for different masses can be easily done by following the above rules.

We can solve equations for densities (4) with conditions (5), (6) analytically in two limit cases for weak separation when $\Delta = u_{12}/\sqrt{u_1 u_2} - 1 \ll 1$, and strong separation when $\Delta \gg 1$. Notice, that both parameters u_{12} and Δ are not affected by the above procedure of mass difference excluding.

II. WEAK SEPARATION

Let us consider weak separation when condition $\Delta = u_{12}/\sqrt{u_1 u_2} - 1 \ll 1$ is satisfied. We expect that in the simplest case when $u_1 = u_2$ the whole density of a gas $n(z) = n_1(z) + n_2(z)$ is approximately constant. That is why, to find the solution it is natural instead of n_1 , n_2 to introduce other quantities and solve equations (4) using small parameter Δ . Consider the functions:

$$\begin{aligned} \rho &= (u_1/u_2)^{1/4} n_1 + (u_2/u_1)^{1/4} n_2, \\ g &= [(u_1/u_2)^{1/4} n_1 - (u_2/u_1)^{1/4} n_2] / \rho \end{aligned} \quad (8)$$

Conditions (5), (6) give us simple asymptotic behavior of these functions:

$$\begin{aligned} \rho(z \rightarrow \pm\infty) &\rightarrow \rho_0 = \sqrt{n_{10} n_{20}}, \\ g(z \rightarrow \pm\infty) &\rightarrow \pm 1 \end{aligned} \quad (9)$$

Densities of condensates “1” and “2” are easily obtained if functions ρ and g are known:

$$\begin{aligned} n_1 &= (u_2/u_1)^{1/4} \rho [1 + g]/2, \\ n_2 &= (u_1/u_2)^{1/4} \rho [1 - g]/2 \end{aligned} \quad (10)$$

It is straightforward to derive equations for ρ and g :

$$\begin{aligned} \frac{\sqrt{\rho}''}{\sqrt{\rho}} - \frac{\rho}{\rho_0} [1 + \alpha g + \frac{\Delta}{2}(1 - g^2)] &= \frac{g'^2}{4(1 - g^2)} - 1 - \alpha g, \\ \frac{g''}{1 - g^2} + \frac{2\sqrt{\rho}' g'}{\sqrt{\rho}(1 - g^2)} + \frac{g g'^2}{(1 - g^2)^2} + 2\alpha &= \frac{\rho}{\rho_0} [2\alpha + \Delta(\alpha - g)] \end{aligned} \quad (11)$$

Here, $f' = \xi_0 \frac{df}{dz}$, and we also introduced $\alpha = \frac{\sqrt{u_1} - \sqrt{u_2}}{\sqrt{u_1} + \sqrt{u_2}}$ and $1/\xi_0^2 = m(u_1 u_2)^{1/4} (\sqrt{u_1} + \sqrt{u_2}) \rho_0 / \hbar^2$.

To find asymptotic solutions of equations (11) in the case of $\Delta \ll 1$ we use iteration method. Let us suppose that terms with derivatives of ρ are much smaller than the others. We will justify this assumption in the end of calculations.

Neglecting terms with derivatives of ρ , from equations (8) we get

$$\begin{aligned} \rho/\rho_0 &= 1 - \frac{g'^2 + 2\Delta(1 - g^2)^2}{4(1 - g^2)(1 + \alpha g)}, \\ \frac{g''}{1 - g^2} + \frac{2g + \alpha(1 + g^2)}{2(1 - g^2)(1 + \alpha g)} g'^2 + \frac{\Delta(1 - \alpha^2)g}{1 + \alpha g} &= 0 \end{aligned} \quad (12)$$

The equation for g can be solved by substitution $g' = f$, so taking into account conditions (9) we get that g is the solution of the equation

$$g' = \sqrt{\Delta(1 - \alpha^2)} \frac{(1 - g^2)}{\sqrt{1 + \alpha g}} \quad (13)$$

At this point it is clear that assumptions we made were correct. Namely, we obtain that $\sqrt{\rho}' \propto \Delta^{3/2}$, $\sqrt{\rho}'' \propto \Delta^2$, $g' \propto \sqrt{\Delta}$, so neglected terms by Δ times less than the others.

Now we can write down solutions for both ρ and g in the parametric form

$$\begin{aligned} 1 - \frac{\rho}{\rho_0} &= \frac{\Delta}{4} \frac{1 - g^2}{(1 + \alpha g)^2} [3 - \alpha^2 + 2\alpha g], \\ \frac{z - z_0}{\xi_0 / \sqrt{\Delta(1 - \alpha^2)}} &= \frac{\sqrt{1 + \alpha}}{2} \ln \left| \frac{\sqrt{1 + \alpha g} + \sqrt{1 + \alpha}}{\sqrt{1 + \alpha g} - \sqrt{1 + \alpha}} \right| \\ &\quad - \frac{\sqrt{1 - \alpha}}{2} \ln \left| \frac{\sqrt{1 + \alpha g} + \sqrt{1 - \alpha}}{\sqrt{1 + \alpha g} - \sqrt{1 - \alpha}} \right| \end{aligned} \quad (14)$$

Here, the second equation is the solution of (13). In these expressions an arbitrary constant z_0 defines the position of the boundary, and in the case of finite geometry with given average number of particles in condensates can be obtained from the condition of equal pressures (6). There is also physically obvious symmetry $z - z_0 \rightarrow z_0 - z$, $g \rightarrow -g$, $\alpha \rightarrow -\alpha$.

Finally, the densities of condensates have the following form:

$$\begin{aligned} n_1 &= \frac{n_{10}}{2} \left(1 - \frac{\Delta}{4} \frac{1 - g^2}{(1 + \alpha g)^2} [3 - \alpha^2 + 2\alpha g] \right) [1 + g], \\ n_2 &= \frac{n_{20}}{2} \left(1 - \frac{\Delta}{4} \frac{1 - g^2}{(1 + \alpha g)^2} [3 - \alpha^2 + 2\alpha g] \right) [1 - g] \end{aligned} \quad (15)$$

With the parametric equation for g , these densities are the main result of the paper. On fig With the use of the described method, it is possible to derive expressions for densities in the next orders of the small parameter Δ in the form of asymptotic series. As follows from the above estimations of neglected terms in equations (11) the next order is proportional to Δ^2 . The typical dependence of densities on the distance from the boundary is shown in Fig. 1 and Fig. 2.

It is interesting to notice that the whole density $n = n_1 + n_2$ at some values of parameters has a well on the boundary as shown in Fig. 1. In the case of weak separation we see that it is broad and shallow. This result is a consequence of the interparticle interactions. The interparticle potential for each condensate acts in some sense as a wall, so we expect that probability of a particle to be close to the boundary decreases, which means lower density. The existence of such a well allows to consider the possibility of confining of a particle of another sort in the vicinity of the boundary, which is discussed below.

Although, we can not further simplify solutions (14), it is interesting to derive asymptotic behavior of these functions in particular limits:

$$\begin{aligned} 1 - g(z \rightarrow +\infty) &\propto \exp\left[-\frac{2z\sqrt{\Delta}}{\xi_2}\right], \\ 1 + g(z \rightarrow -\infty) &\propto \exp\left[\frac{2z\sqrt{\Delta}}{\xi_1}\right], \\ g(z \rightarrow z_0) &\rightarrow \frac{(z-z_0)\sqrt{\Delta(1-\alpha^2)}}{\xi_0} \end{aligned} \quad (16)$$

Here, $\xi_i = \hbar/\sqrt{2m_i\mu_i}$ – correlation lengths of condensates, defined by chemical potential μ_1 and μ_2 , and we used (7) to include mass difference. The asymptotic behavior far from the boundary of each condensate is defined by its correlation length. As we see, the size of the boundary can be approximated as $d \sim (\xi_1 + \xi_2)/\sqrt{\Delta}$ and for $\Delta \ll 1$ appears to be much larger than correlation lengths.

Solutions have the simplest form when $\alpha = 0$:

$$\begin{aligned} n_1(z) &= \frac{n_0}{2} \left(1 - \frac{3\Delta}{4 \cosh^2[\frac{z\sqrt{\Delta}}{\xi_0}]} \right) \left(1 + \tanh\left[\frac{z\sqrt{\Delta}}{\xi_0}\right] \right), \\ n_2(z) &= \frac{n_0}{2} \left(1 - \frac{3\Delta}{4 \cosh^2[\frac{z\sqrt{\Delta}}{\xi_0}]} \right) \left(1 - \tanh\left[\frac{z\sqrt{\Delta}}{\xi_0}\right] \right) \end{aligned} \quad (17)$$

Here, we choose $z_0 = 0$.

III. STRONG SEPARATION

To analyze the case of strong separation $\Delta = u_{12}/\sqrt{u_1 u_2} \gg 1$ (we use the same notation but for another quantity) we start from density equations (4). In this case we expect that density on the boundary will be approximately zero because interparticle interactions make it almost impossible for one condensate to penetrate inside the other. To estimate the density of condensates on the boundary we can use the fact that second derivatives of wave functions should be approximately

zero there. That makes system (4) a set of two linear equations with solution:

$$\begin{aligned} n_{1B} &= \frac{n_{10}}{\Delta+1} \approx \frac{n_{10}}{\Delta} \ll n_{10}, \\ n_{2B} &= \frac{n_{20}}{\Delta+1} \approx \frac{n_{20}}{\Delta} \ll n_{20} \end{aligned} \quad (18)$$

This allows us in the zero approximation to use simple conditions for the densities $n_1(z \leq 0) = n_2(z \geq 0) = 0$. Then equations (4) have simple form:

$$\begin{aligned} \mu_1 &= -\frac{\hbar^2}{2m_1\sqrt{n_1}} \frac{d^2}{dz^2} \sqrt{n_1} + u_1 n_1, \quad \text{for } z \geq 0, \\ \mu_2 &= -\frac{\hbar^2}{2m_2\sqrt{n_2}} \frac{d^2}{dz^2} \sqrt{n_2} + u_2 n_2, \quad \text{for } z \leq 0 \end{aligned} \quad (19)$$

Solutions are easily obtained:

$$\begin{aligned} n_1(z \geq 0) &= n_{10} \tanh^2\left[\frac{z}{\sqrt{2}\xi_1}\right], \\ n_2(z \leq 0) &= n_{20} \tanh^2\left[\frac{z}{\sqrt{2}\xi_2}\right] \end{aligned} \quad (20)$$

Here, n_{10} and n_{20} are connected by the condition of equal pressures (6), and we choose the position of the boundary at $z = 0$.

The size of the boundary in this case is approximately $d \approx 2\sqrt{2}(\xi_1 + \xi_2)$. The dependence of the densities on the distance from the boundary is shown in Fig. 3. As in the previous section, we see that there is again a well in the whole density but in the case of strong separation it becomes narrower and deeper in comparison with the one for weak separation.

IV. ONE-PARTICLE EXCITATIONS ON THE BOUNDARY

The existence of a well in the whole density allows to consider confining of a particle of another sort in the vicinity of the boundary. As a general property of a quantum-mechanical motion in a one-dimensional well there always exists such a confined state. As an example, we consider the simplest case when $\alpha = 0$ and a particle of another sort interacts with both condensates repulsively with the same constant λ . The Schrödinger equation for the wave function of a particle with the mass M has the form:

$$\left[-\frac{\hbar^2}{2M} \frac{d^2}{dz^2} + \lambda n(z) \right] \phi = E \phi \quad (21)$$

We can solve equation (21) for weak and strong separation cases simultaneously. It has universal form

$$\frac{d^2 \phi}{dz^2} + \frac{2M}{\hbar^2} \left(\epsilon + \frac{U_0}{\cosh^2[\beta z]} \right) \phi = 0, \quad (22)$$

where $\epsilon = E - \lambda n_0$, and $U_0 = 3\Delta\lambda n_0/4$, $\beta = \sqrt{\Delta}/\xi_0$ for weak; and $U_0 = \lambda n_0$, $\beta = 1/(\sqrt{2}\xi_0)$ for strong separation. The spectrum of energy ϵ is well-known:

$$\begin{aligned}\epsilon_j &= -\mu \frac{\Delta m}{4M} \left[-(1+2j) + \sqrt{1 + 3 \frac{M\lambda}{mu}} \right]^2 \text{ "weak",} \\ \epsilon_j &= -\mu \frac{m}{8M} \left[-(1+2j) + \sqrt{1 + 8 \frac{M\lambda}{mu}} \right]^2 \text{ "strong"}\end{aligned}\quad (23)$$

where $\mu = un_0$ – chemical potential, $j = 0, 1, \dots$ and the condition that an expression in [...] is positive defines the upper limit for j . There is always at least one state with $j = 0$.

V. SURFACE WAVES ON THE BOUNDARY

There is also another type of excitations associated with the boundary. As we see condensate density distributions give rise to nonzero surface tension which was previously analyzed in [8], and we use the expression for surface tension derived there:

$$\sigma = \frac{1}{2} \int_{-\infty}^{+\infty} dz \sum_{i=1,2} \frac{\hbar^2}{2m_i} \left(\frac{d\sqrt{n_i}}{dz} \right)^2 \quad (24)$$

Substituting expressions (15) for densities in the case of weak separation and taking into account only first nonzero order in Δ we obtain for surface tension

$$\sigma_w = \frac{P\sqrt{\Delta}}{6} \left[\begin{aligned} &(\xi_1 + \xi_2) \frac{2\alpha^2 - 1 + \sqrt{1 - \alpha^2}}{\alpha^2} \\ & - (\xi_1 - \xi_2) \frac{1 - \sqrt{1 - \alpha^2}}{\alpha} \end{aligned} \right], \quad (25)$$

where $\alpha = (m_1\sqrt{u_1} - m_2\sqrt{u_2}) / (m_1\sqrt{u_1} + m_2\sqrt{u_2})$ and we used (7) to include mass difference; ξ_1, ξ_2 – correlation lengths of condensates; P – the pressure given by (6). When $\alpha = 0$, $\sigma_w = P\sqrt{\Delta}(\xi_1 + \xi_2)/4$

In the case of strong separation we use expressions (20) to get surface tension

$$\sigma_s = \frac{P\sqrt{2}}{3}(\xi_1 + \xi_2) \quad (26)$$

Let us notice that expressions (25) and (26) differ from the ones obtained in [8]. Although, in qualitative sense our expressions coincide with those of [8], using our method of solution we can get a general expression applicable for variety of parameters, and for example retrieve the correct numerical factor for the case considered in [8]. As follows from the general expression for surface tension (24), we need to know the behavior of densities not only far but also in the vicinity of the boundary. That is why, estimate character of expressions for densities in [8] could only give qualitative answer for surface tension.

For velocities smaller than the speed of sound we can consider gas as incompressible, so it is possible to write down the usual hydrodynamic equations and find dispersion relation of surface waves on the boundary associated with surface tension. Suppose that we have condensates in a “box” and the position of the boundary is defined by the distances L_1 and L_2 from the “box” walls, where

labels “1”, “2” correspond to the side with the same condensate, and the box sizes for other directions are L_x, L_y . Taking into account the fact that velocities are zero on the walls we get the dispersion relation

$$\omega^2(k) = \frac{\sigma k^3}{\rho_1 \coth(kL_1) + \rho_2 \coth(kL_2)}, \quad (27)$$

where σ is the surface tension; $\rho_{10} = m_1 n_{10}$, $\rho_{20} = m_2 n_{20}$ – mass densities; wave vector along the boundary $k = \sqrt{(\pi n_x/L_x)^2 + (\pi n_y/L_y)^2}$, $n_x, n_y = 0, 1, \dots$, not $n_x = n_y = 0$; L_1, L_2 – sizes of condensates in the direction perpendicular to the boundary. We assume that L_1, L_2 are much larger than the size of the boundary d .

For dispersion relation we consider two limit cases corresponding to wavelengths $\lambda \ll L_i$ ($kL_i \gg 1$) and $\lambda \gg L_i$ ($kL_i \ll 1$). Let us notice that the second case is possible only if $L_i \ll L_{x,y}$.

In the first case we obtain

$$\omega(k) = \left(\frac{\sigma}{\rho_{10} + \rho_{20}} \right)^{1/2} k^{3/2} \quad (28)$$

In the long wavelength limit we get

$$\omega(k) = \left(\frac{\sigma L_1 L_2}{\rho_{10} L_2 + \rho_{20} L_1} \right)^{1/2} k^2 \quad (29)$$

As we see, for weak separation $\omega \propto \Delta^{1/4}$, and surface waves are relatively “soft” in that case, which enables to consider them as a dissipative channel for other condensate excitations.

VI. CONCLUSION

We performed the analysis of the boundary of two Bose-Einstein condensates interacting repulsively in the limit cases corresponding weak and strong separation at zero temperature.

For weak separation we obtained solutions (15) of two coupled nonlinear Gross-Pitaevskii equations using small parameter Δ . The solutions show that the penetration depth of the condensate “ i ” inside the other is estimated as $\xi_i/\sqrt{\Delta}$, so that the size of the boundary $d \sim (\xi_1 + \xi_2)/\sqrt{\Delta}$ is much larger than correlation lengths, which was obtained experimentally [5]. There is also a well in the full density profile, which is the consequence of the wave function behavior near the boundary. In general, the proposed method of obtaining density distributions for the weak separation case if desired can be expanded to obtain expressions for the next orders of Δ .

We also considered the case of strong condensate separation but restricted ourselves to the zero order approximation. In that case the size of the boundary is $d \sim (\xi_1 + \xi_2)$, and the whole density of gases goes approximately to zero on the boundary.

The existence of the well in the density profile at some parameters allowed us to consider one-particle excitations on the boundary. Using the expressions for densities we found excitation spectrum of a particle in the simplest case when constants of interaction of the particle with condensates are the same, and distribution of densities on the boundary corresponds to $\alpha = 0$. The generalization to other cases can be easily done with the use of expressions (15),(20) for density distributions. Let us notice that the existence of a potential well depends on the interaction constants of a particle with condensates as well as the relation between interaction constants of the condensates. Observation of such confined states can be possible only if temperature are smaller than the potential well.

It was shown that there also exist collective excitations associated with surface tension. The expressions for surface tension were obtained for both weak and strong separation. The dispersion relation for surface waves was analysed in the case where condensates fill finite volumes. The dispersion relation has different forms in two cases corresponding to short- and long-wavelength limit. In the case of weak separation “soft” surface modes can be a dissipative channel of other condensate excitations.

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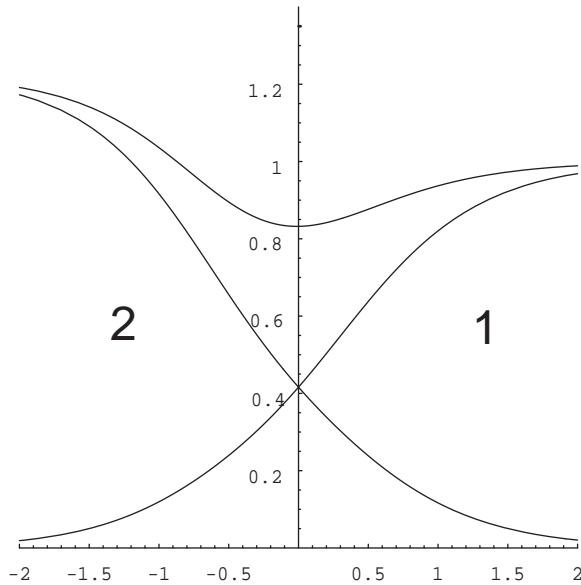


FIG. 1. Distribution of densities in the case of weak separation. $\Delta = 0.3$, $\alpha = 0.1$. Densities are in the scale of n_{10} . Distance is in the scale of $\xi_0/\sqrt{\Delta(1-\alpha^2)}$. There is a well in the whole density profile.

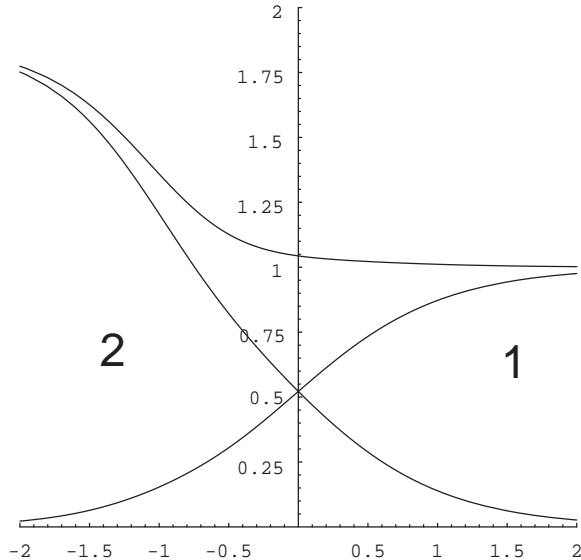


FIG. 2. Distribution of densities in the case of weak separation. $\Delta = 0.3$, $\alpha = 0.3$. Densities are in the scale of n_{10} . Distance is in the scale of $\xi_0/\sqrt{\Delta(1-\alpha^2)}$.

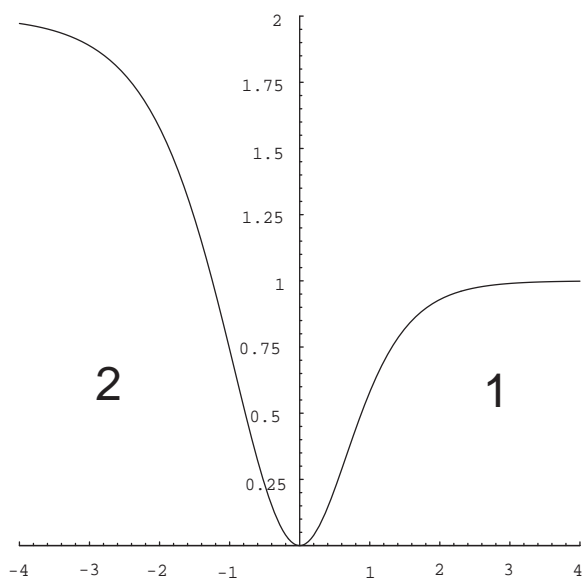


FIG. 3. Distribution of densities in the case of strong separation when $u_1/u_2 = 4$. Densities are in the scale of n_{10} . Distance is in the scale of $\sqrt{2}\xi_1$.